SMART LIGHTING

SMART LIGHTING ENGINEERING RESEARCH CENTER

Lighting Innovation for a Smarter Tomorrow

AC Circuits

T. P. Chow

June 28, 2011

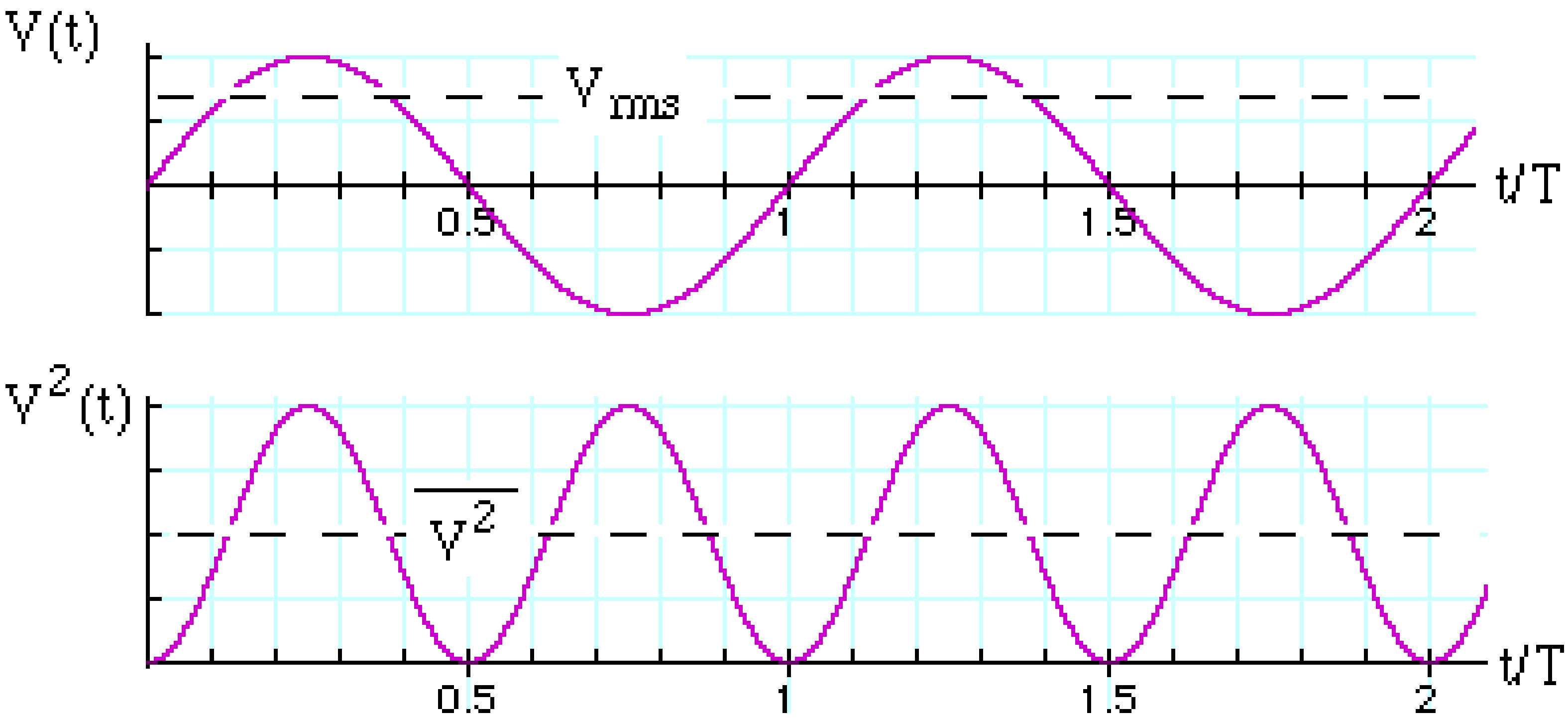
Rensselaer BOSTON UNIVERSITY UNM NYSTAR NSF

AC Circuits

- AC electricity is ubiquitous - not only in the supply of power, but in electronics and signal processing
- In the US, the home socket provides AC 120V, 60Hz (different in Europe, Asia, …)
- 110V is rms value, maximum voltage value is

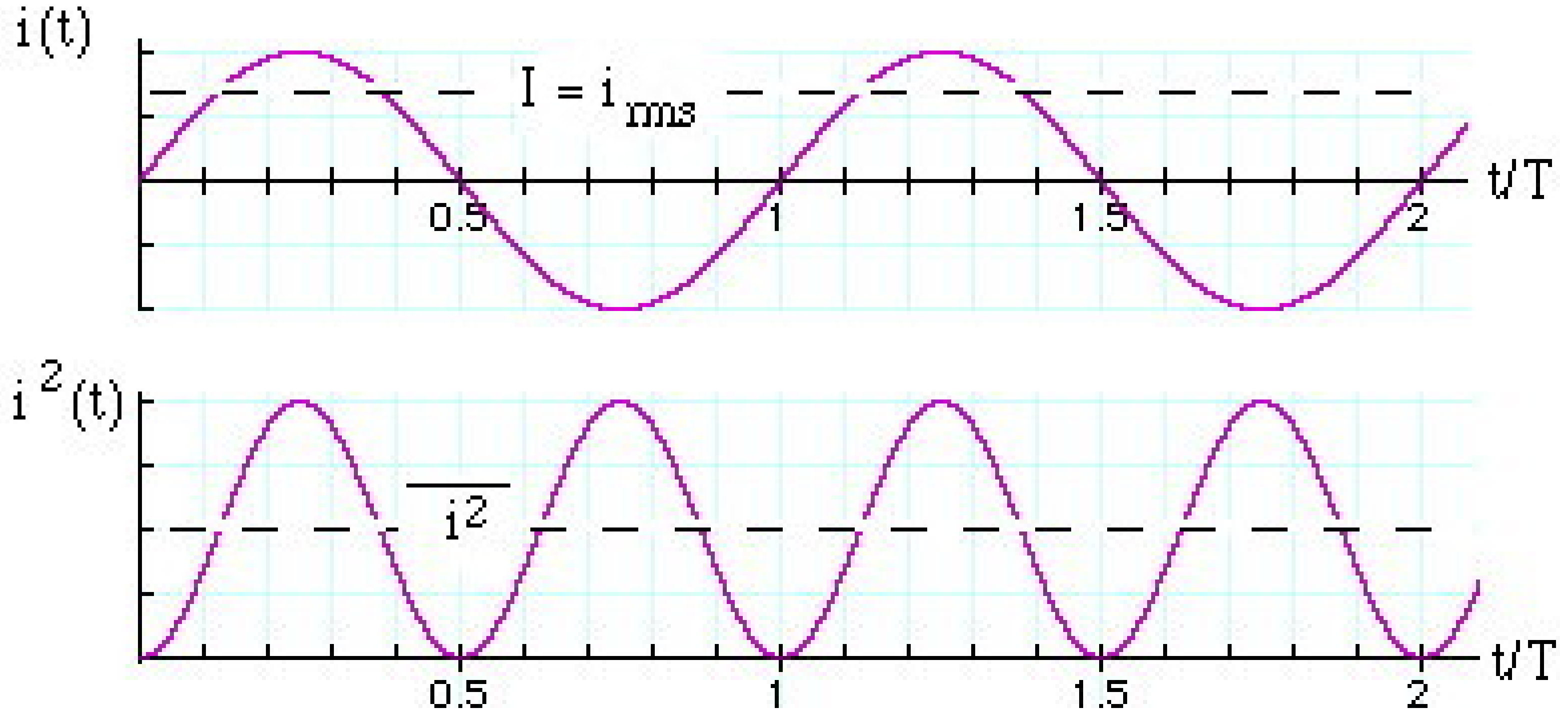
  $V_m = \sqrt{2} \times V_{rms} = 170V$

V_{rms} vs. V_m



$v(t) = V_m \sin(\omega t + \phi)$
where $\omega = 2\pi f$

I_{rms} vs. I_m



$i(t) = I_m \sin(\omega t + \phi)$

where $\omega = 2\pi f$

Power

- Instantaneous Power

  $p(t) = i(t)\, v(t) = v^2/R = i^2R$
- Mean Power Delivered (to resistors)

  $P = (1/T) \int p(t)\, dt$

  $P = R\, I_m^{\,2} / 2 = R\, I^2 = I\, V = V^2 / R$

  where $I \cong I_{rms} = I_m / \sqrt{2}$ and $V \cong V_{rms} = V_m / \sqrt{2}$
- Mean Power Delivered to (ideal) inductors and capacitors is zero

AC Circuits

- http://www.animations.physics.unsw.edu.au/jw/AC.html

AC Circuits

- In DC Circuits We Saw
  - Resistors – Ohm's Law V=RI. Resistors dissipate energy (they turn electrical energy into heat)
  - Diodes – Complicated Exponential Function (control the direction of current flow)
- There are two other passive circuit elements: Inductors and Capacitors
  - Capacitors store energy as charge $W_C = C\frac{V^2}{2} = \frac{Q^2}{2C}$
  - Inductors store energy as current

$$W_L = L\frac{I^2}{2}$$

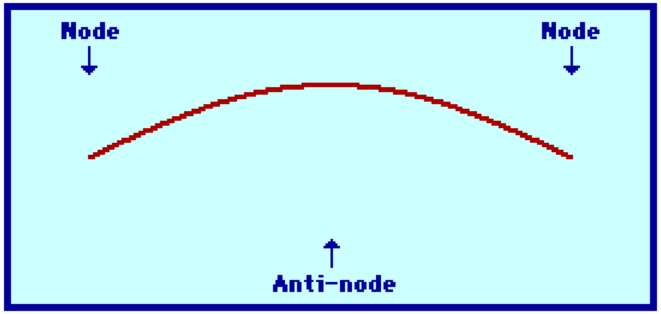
Energy Storage

- Note that the form of the capacitor and inductor formulas is the same as for the kinetic and potential energy for a mass on a spring. (k is the spring constant)

$$W_{KE} = m\frac{v^2}{2} \qquad W_{PE} = k\frac{x^2}{2}$$

- The mass at the end of a spring will oscillate as energy is traded between KE and PE. The same thing happens with C & L

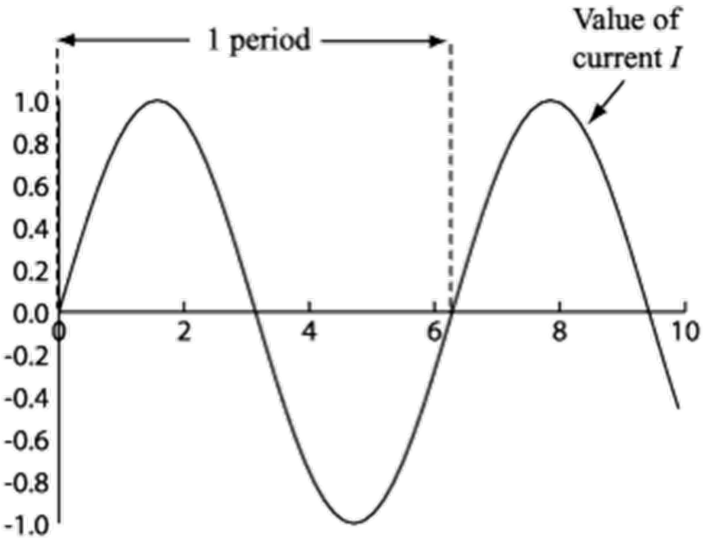
Periodically Oscillating Systems



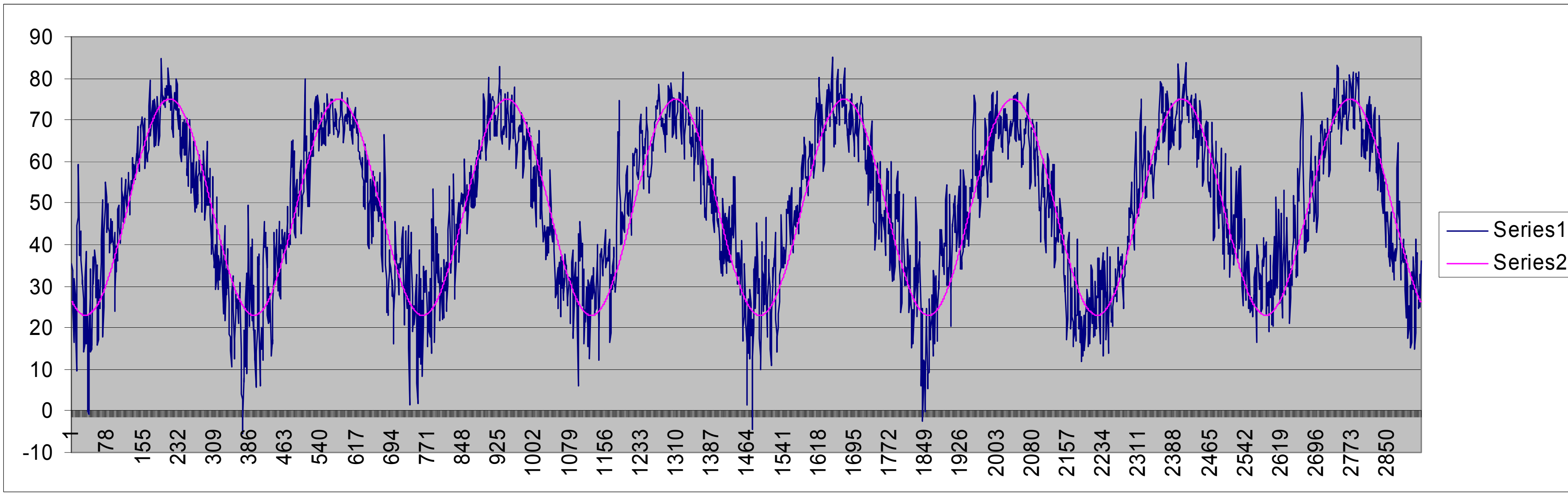
- Pendulum
- Bouncing ball
- Vibrating string (stringed instrument)
- Circular motion (wheel)
- Cantilever beam (tuning fork)

Other Periodic Phenomena

- Daily cycle of solar energy
- Annual cycle of solar energy
- Daily temperature cycle
- Annual temperature cycle
- Monthly bank balance cycle
- Electronic clock pulse trains
- Line voltage and current



Daily Average Temperature Albany-Schenectady-Troy



- Data (blue) covers 1995-2002
- Note the sinusoid (pink) fit to the data

Using Models

- You should use a model that you understand and/or know how to properly apply
- To use it properly
  - Check for plausibility of predicted values (ballpark test). Are the values in a reasonable range?
  - Check the rate of changes in the values (checking slope of plot).
  - Are the most basic things satisfied?
    - Conservation of energy, power, current, etc.
- Developing a qualitative understanding of phenomena will help when you use a more sophisticated approach in the future.

Capacitors

- http://www.animations.physics.unsw.edu.au/jw/AC.html

Return to Capacitors & Inductors

$$W_L = L\frac{I^2}{2} \qquad W_C = C\frac{V^2}{2}$$

- A capacitor can also do work. We have observed how charging something up (like a balloon in the winter) can produce a force on an insulator (the balloon sticks to the wall). We have also seen how charged up objects (Styrofoam balls or cups) stick to our hands in winter
- Since the energy is stored in the charge or voltage and since we cannot instantaneously change the energy of anything, we conclude that it is not possible to instantaneously change the charge or voltage stored in a capacitor. That is, V cannot jump from one value to another in no time.

Inductors

- http://www.animations.physics.unsw.edu.au/jw/AC.html

Return to Capacitors & Inductors

$$W_L = L\frac{I^2}{2}$$

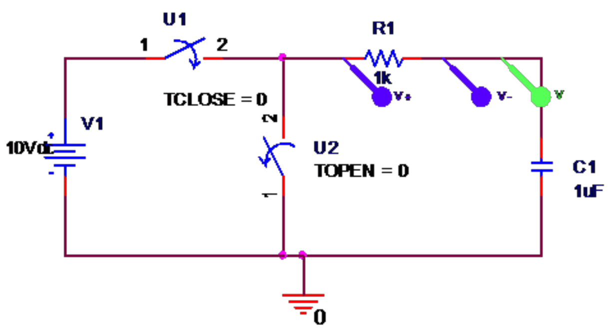
$$W_C = C\frac{V^2}{2}$$

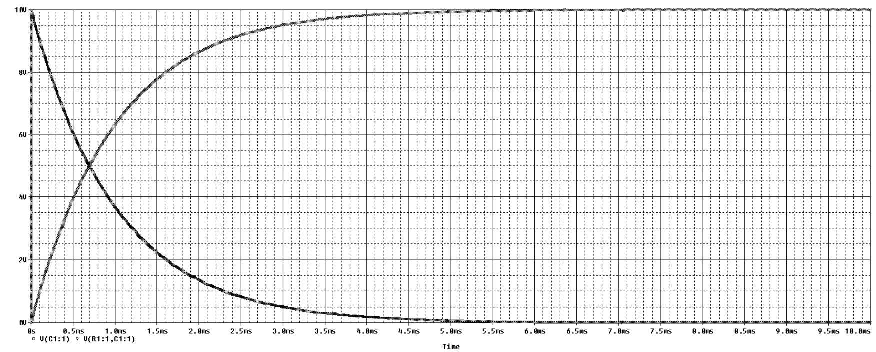
- An inductor is an electromagnet. We know it can do work because we have see it produce a force on magnetic materials.
- Since the energy is stored in the current and since we cannot instantaneously change the energy of anything, we conclude that it is not possible to instantaneously change the current in an inductor. That is, I cannot jump from one value to another in no time.

Looking at Some Examples

- First, we will address charging a capacitor, using the results from a computer simulation tool – Pspice
- We will need to use Ohm's Law for this, which stated in words is: the voltage drop across a resistor divided by the resistance is equal to the current flowing through the resistor.
- Example: a 10V battery is instantaneously switched across a series combination of a resistor and a capacitor.

Charging a Capacitor



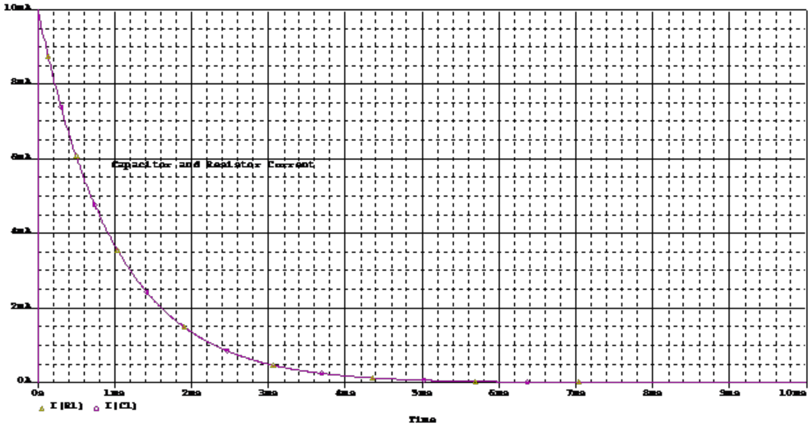


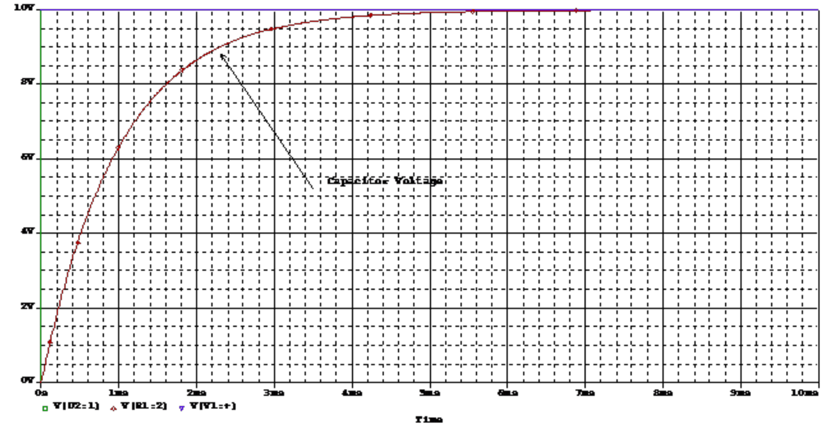
- Capacitor C1 is charged up by current flowing through R1

$$I = \frac{V1 - V_{CAPACITOR}}{R1} = \frac{10 - V_{CAPACITOR}}{1k}$$

- As the capacitor charges up, its voltage increases and the current charging it decreases (the voltage across the resistor decreases), resulting in the charging rate shown

Charging a Capacitor





- Capacitor Current $$I = I_o e^{-t/\tau}$$
- Capacitor Voltage $$V = V_o\left(1 - e^{-t/\tau}\right)$$
- Where the time constant $\tau = RC = R1 \cdot C1 = 1ms$

2 Minute True-False Test

- If $C_1 < C_2$, for a fixed charging current, it will take longer to charge C_1 than C_2
- If $R_1 < R_2$, for a fixed charging voltage, it will take longer to charge a given capacitor C through R_1 than R_2
- When a capacitor C is connected to a battery through a resistor R, the charging current will be a maximum at the moment the connection is made and decays after that.

What About Inductors?

For an inductor, the current cannot change instantaneously but the voltage can. You have all observed what happens when you try to change the current in an inductor very quickly.

- Inductor Voltage $$V = V_o e^{-t/\tau}$$
- Inductor Current $$I = I_o\left(1 - e^{-t/\tau}\right)$$
- Where the time constant $$\tau = L/R$$

Voltage & Current in L & C

- The current producing an increase in the charge on a capacitor is $I = \Delta Q / \Delta t$ where ΔQ is the increase in charge and Δt is the time the current flows.
- The voltage and charge on a capacitor are proportional. More charge means more voltage. In fact, the capacitance is the constant of proportionality. $Q = CV$

Voltage & Current in L & C

$$I = \Delta Q / \Delta t = C\Delta V / \Delta t \qquad \Delta V / I = \Delta t / C$$

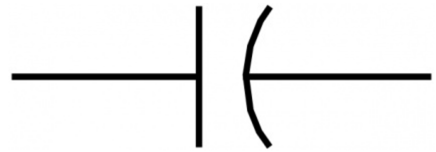
- Since the ratio of voltage to current is resistance (for resistors) we expand the definition to be impedance for all circuit components, but it behaves basically like resistance.
- At very high frequencies, things change quickly so Δt is very small and the impedance is also very small. In fact, in the limit as the frequency goes to infinity, the impedance goes to zero and the capacitor looks like a short circuit. (More on this below)

Voltage & Current in L & C

$$I = \Delta Q / \Delta t = C\Delta V / \Delta t \qquad \Delta V / I = \Delta t / C$$

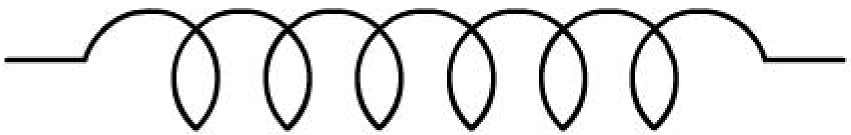
- At very low frequencies, things change slowly so Δt is very large and the impedance is also very large. In fact, in the limit as the frequency goes to zero, the impedance goes to infinity and the capacitor looks like a open circuit. (More on this below)
- From intuition, we can see that these two results make sense (next page)

Voltage & Current in C



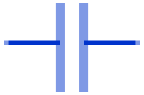
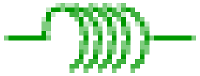
- As reflected by its symbol, a capacitor generally consists of two large area metal plates that are very close to one another, but do not touch.
  - At DC, a capacitor looks like wires that are not connected, so current cannot flow across the space between the plates.
  - At very high frequencies, the plates act like identical radio antennas that are very close to one another. They are so close, in fact, that they couple perfectly and all the energy radiated from one is received by the other and it looks like a short circuit.

Voltage & Current in L

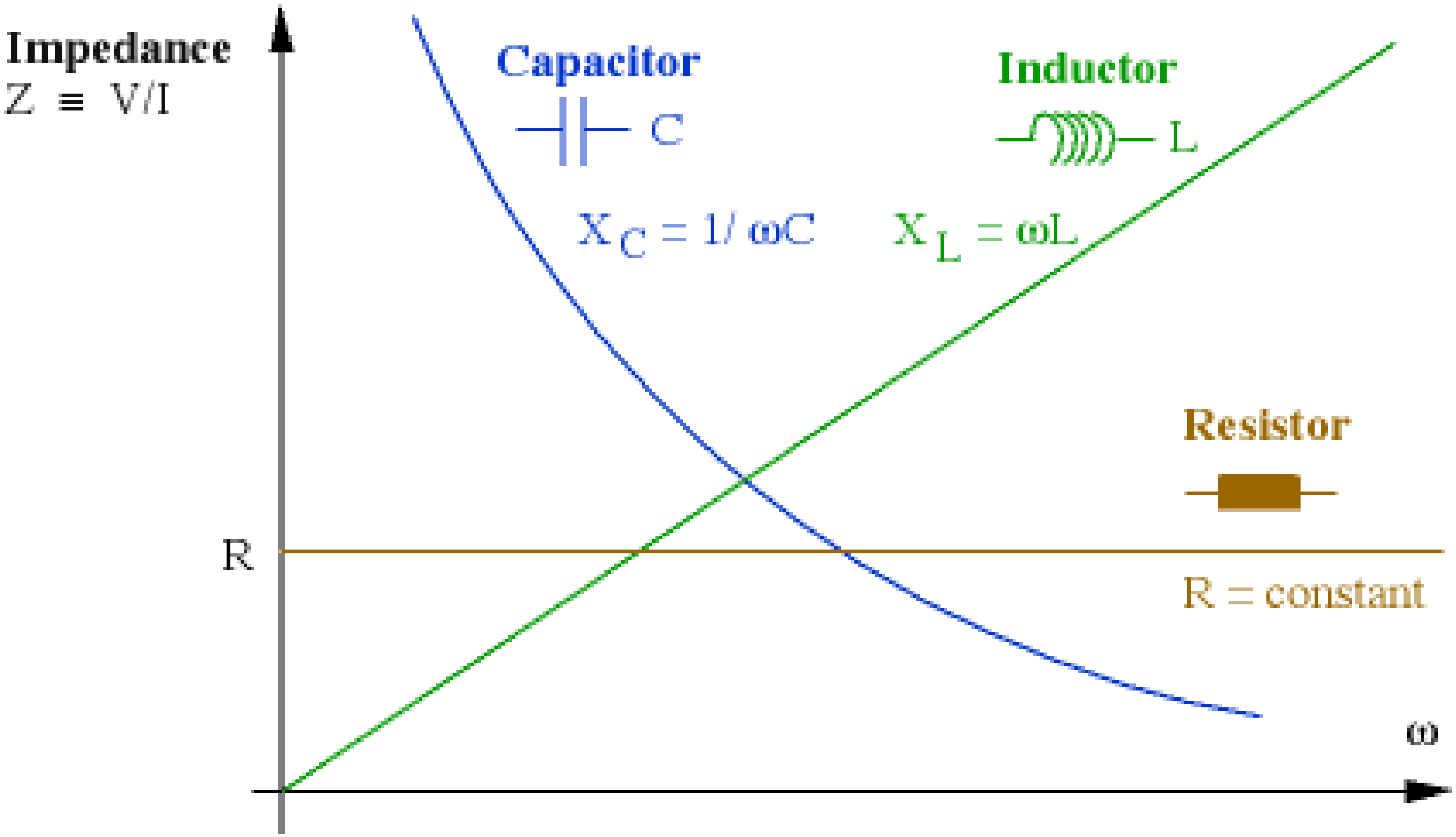


- As reflected by its symbol, an inductor generally consists of large continuous coil of wire, with no breaks in it.
  - At DC, an inductor just looks like a long wire and since the current is not changing, it acts like a short circuit.
  - At very high frequencies, we have to go back to the argument where the current cannot change quickly. $V / \Delta I = L / \Delta t$ and the impedance is very large so it looks like an open circuit.

Impedance

Resistor	Capacitor	Inductor
 R	 C	 L
Resistance	Capacitive reactance	Inductive reactance
$V_R/I = R$	$V_C/I = X_C = \dfrac{1}{\omega C}$	$V_L/I = X_L = \omega L$
V and I in phase	V lags I by $\pi/2$	V leads I by $\pi/2$

Impedance


Impedance
Z ≡ V/I
Capacitor
C
X C = 1/ ωC
Inductor
L
X L = ωL
Resistor
R
R = constant
ω

Capacitors

- Simplest form is two parallel plates separated by a dielectric
- Voltage on a capacitor depends on the amount of charges stored
- Current flowing onto the positive capacitor plate is the rate at which charge is being stored
- $v_C = q / C = (1 / C) \int i \, dt$

Capacitors

- http://www.animations.physics.unsw.edu.au/jw/AC.html

Inductors

- Usually a coil of wire
- Current flowing in an inductor set up a magnetic field, which determines the magnetic flux stored
- Voltage across an inductor is due to its own magnetic field and Faraday's Law
- $\phi_B\ (t) = L\ i(t)$
- $v_L = d\phi_B\ /dt = d/dt\ (Li)$

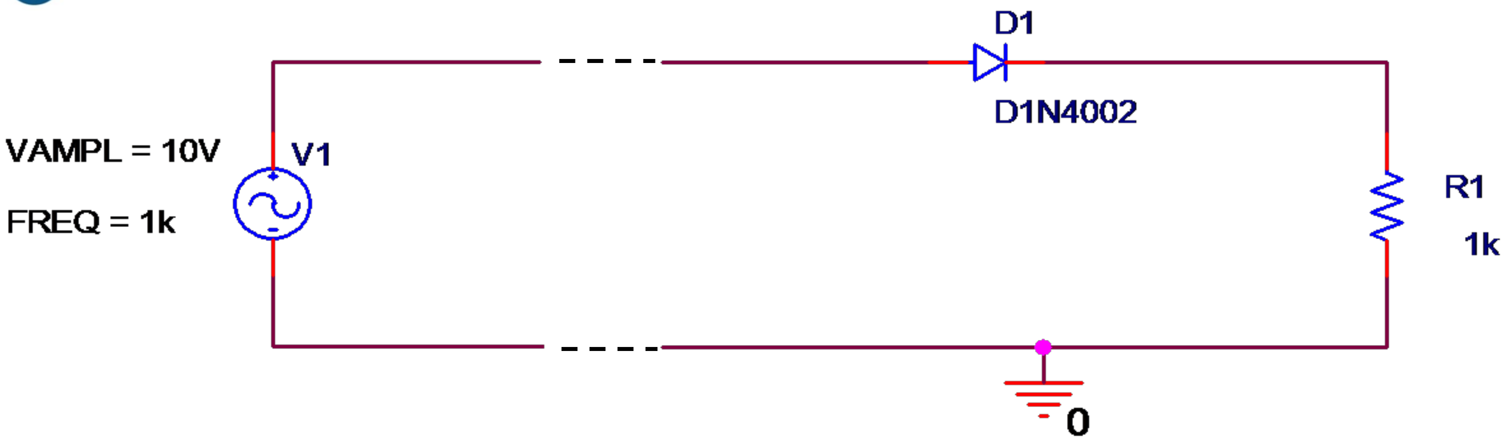
Inductors

- http://www.animations.physics.unsw.edu.au/jw/AC.html

Capacitors and Inductors

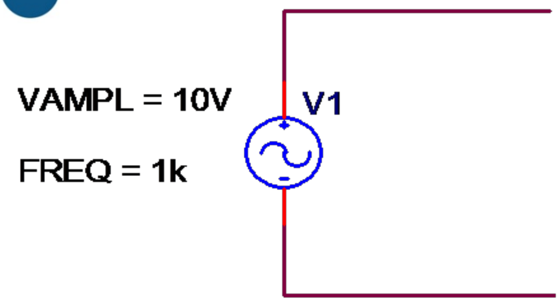
- Can you name capacitors and inductors in your household appliances?

Diodes

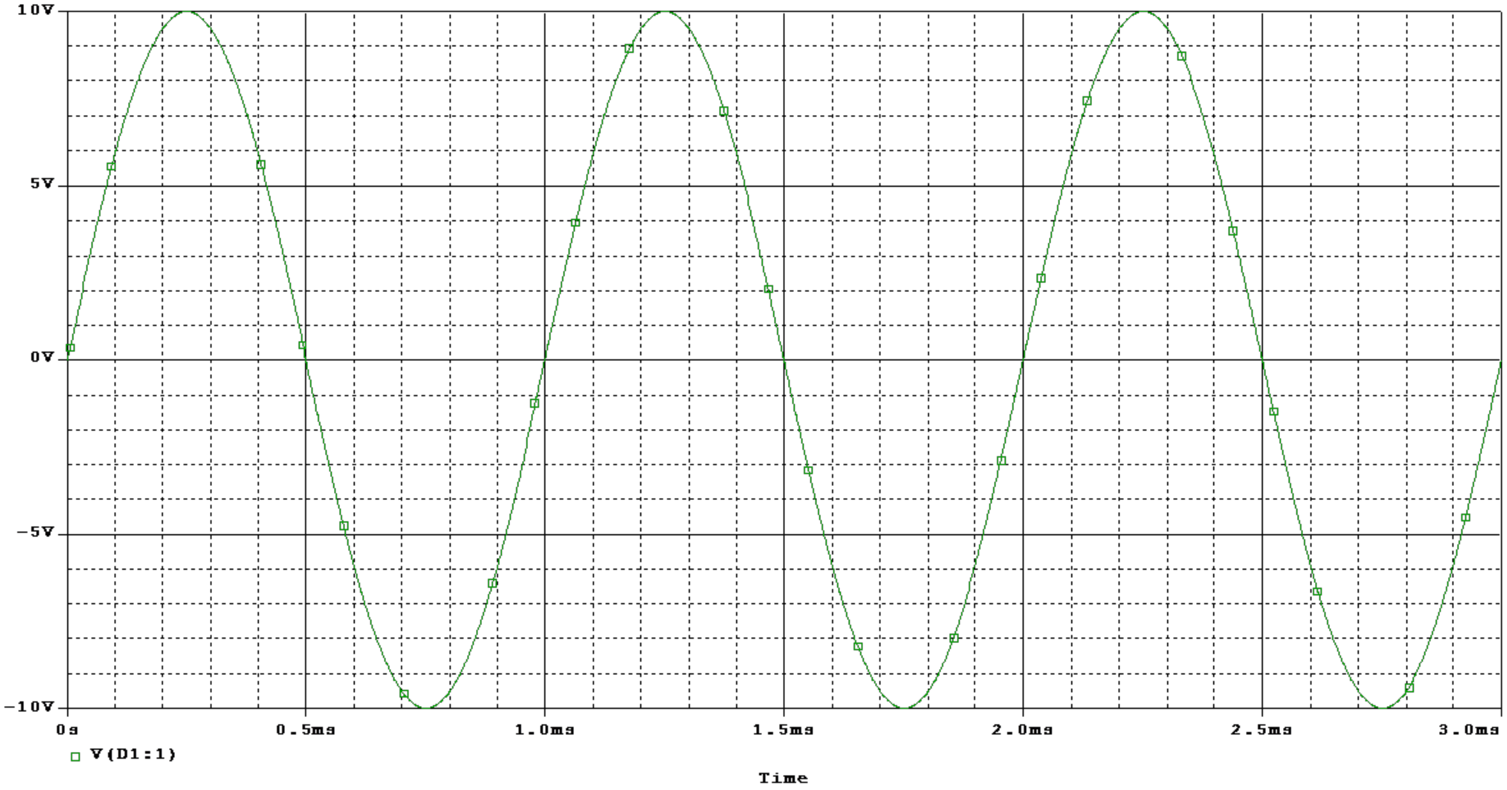


- 10 volt sinusoidal voltage source
- Connect to a resistive load through a diode
  - This combination is called a half-wave rectifier

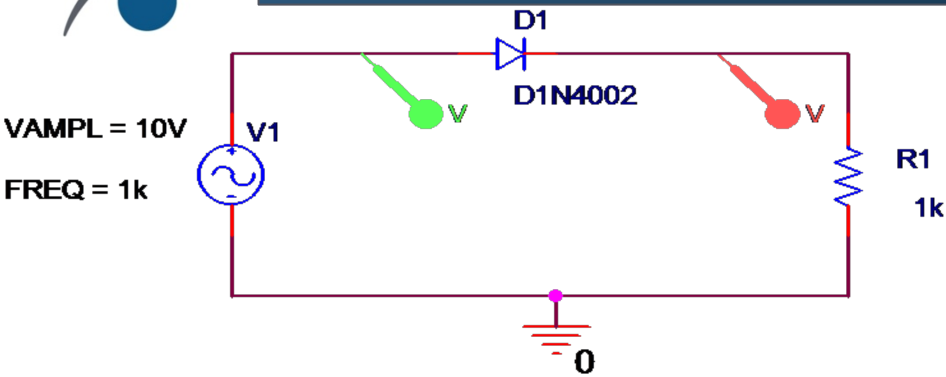
Diodes



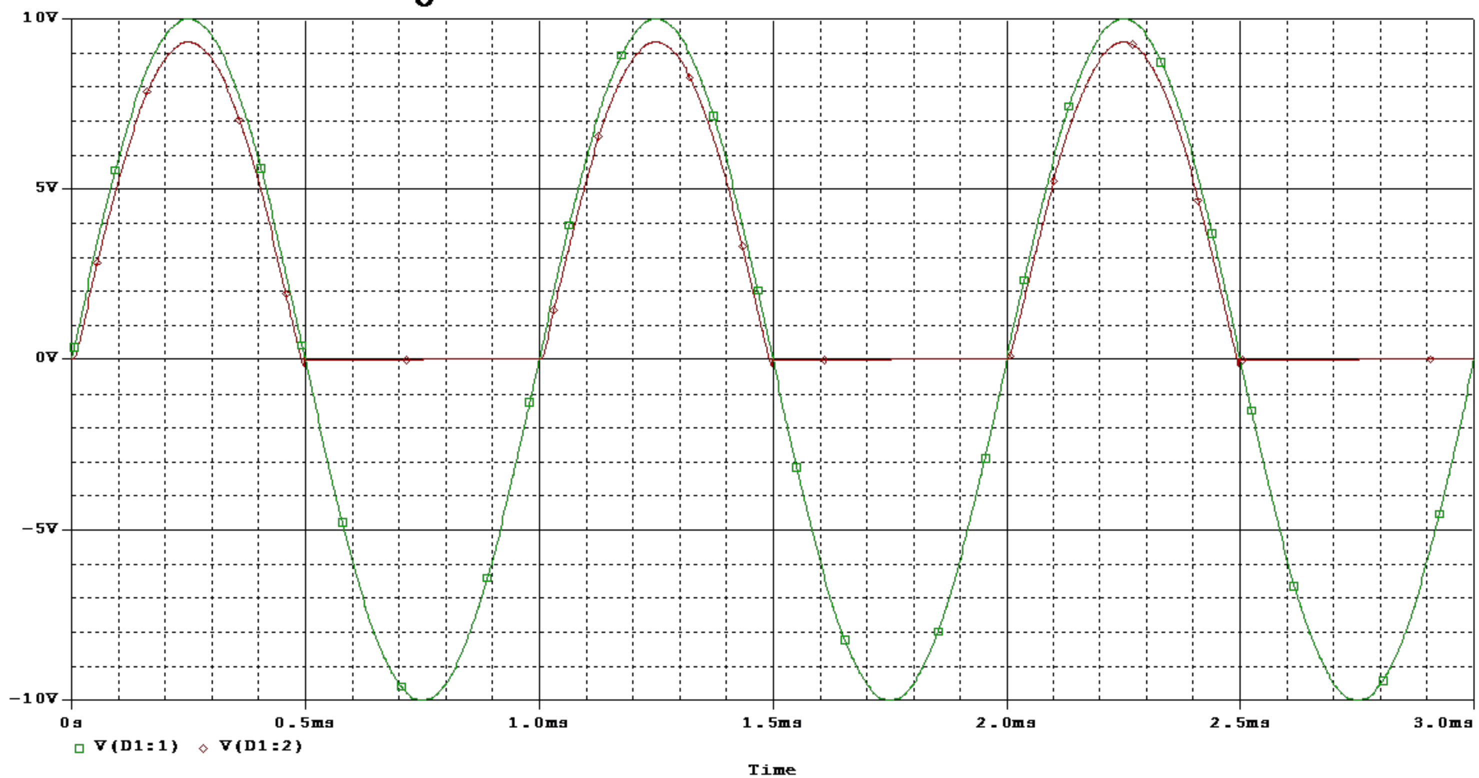
- Sinusoidal Voltage



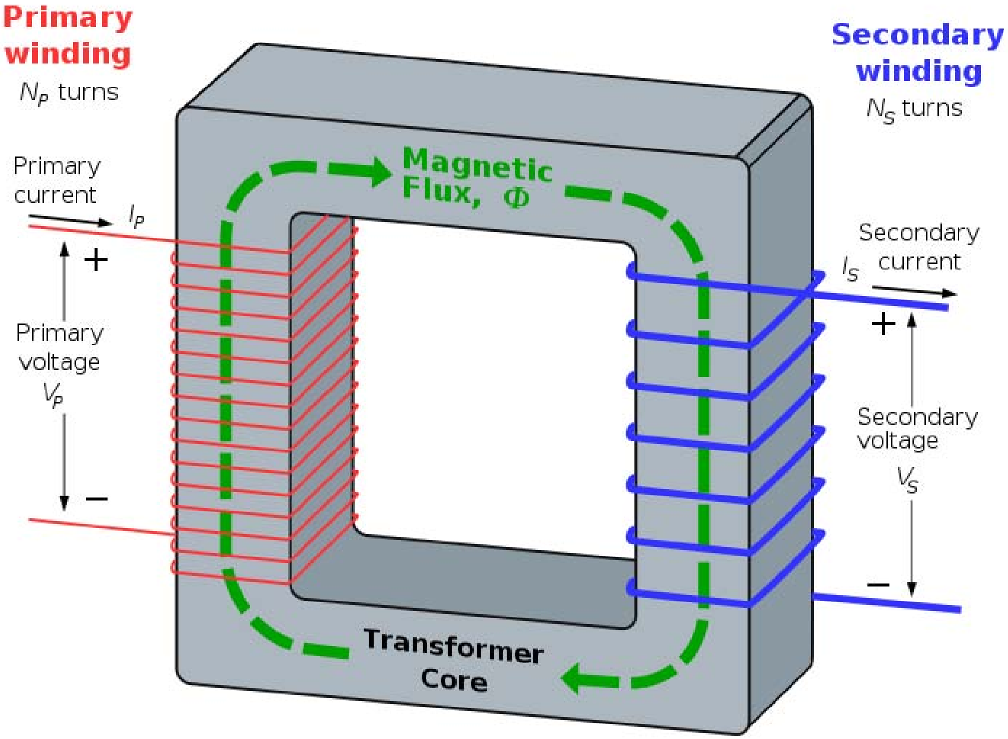
Diodes



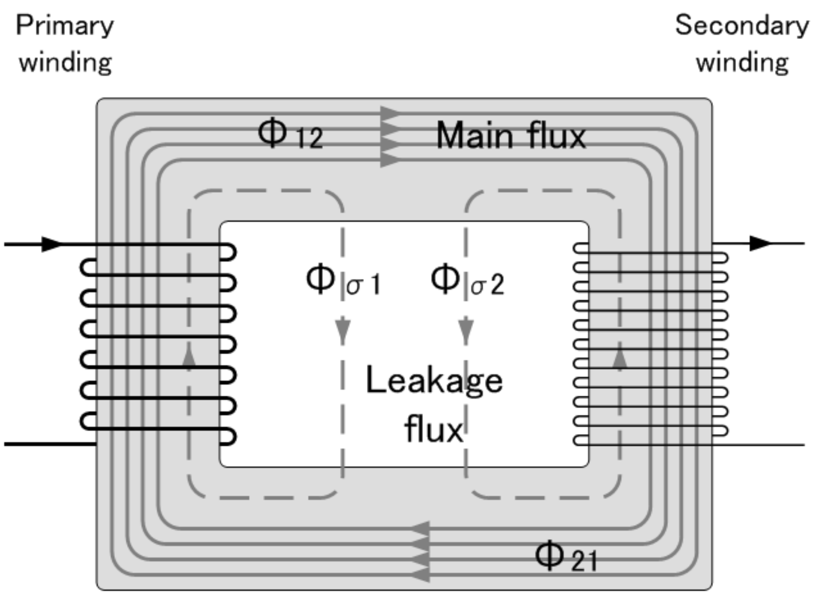
- Half-wave rectifier

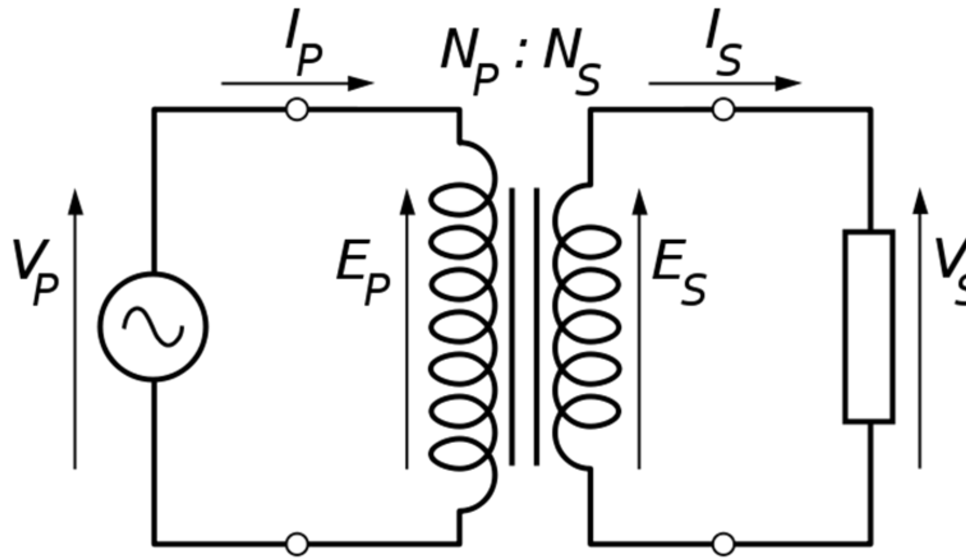


Transformer



Transformer





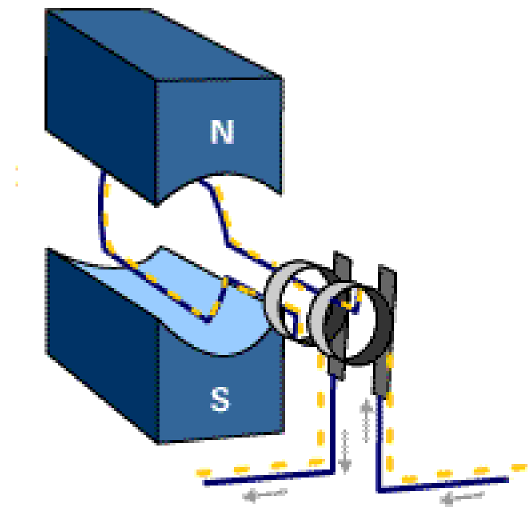
$$\frac{V_P}{V_S} = \frac{I_S}{I_P} = \frac{N_P}{N_S}$$

AC Generator

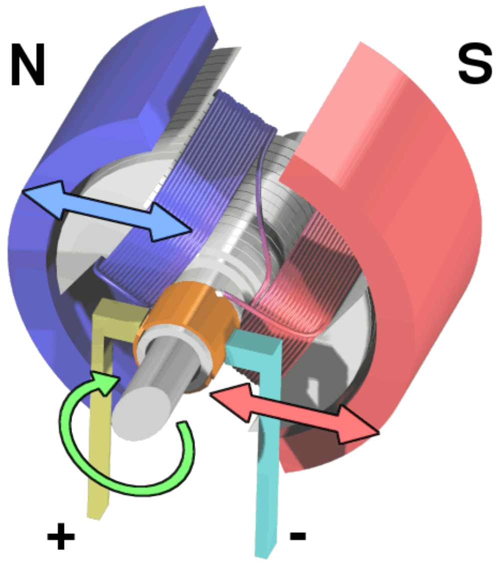
A loop of wire spinning through a magnetic field will create an alternating current. Note: current will flow only if the circuit connected to the generator is complete.

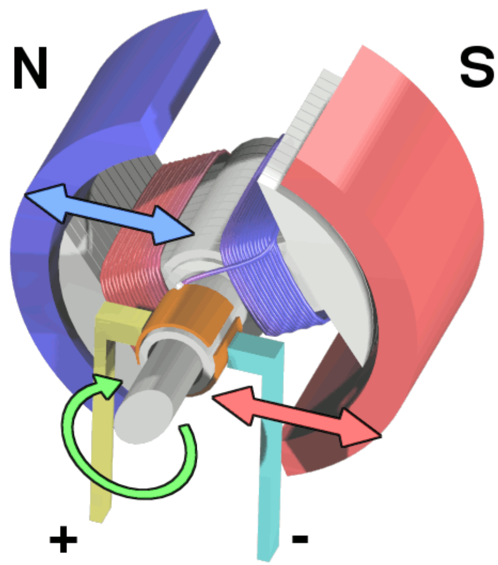
http://www.pbs.org/wgbh/amex/edison/sfeature/acdc_insideacgenerator.html

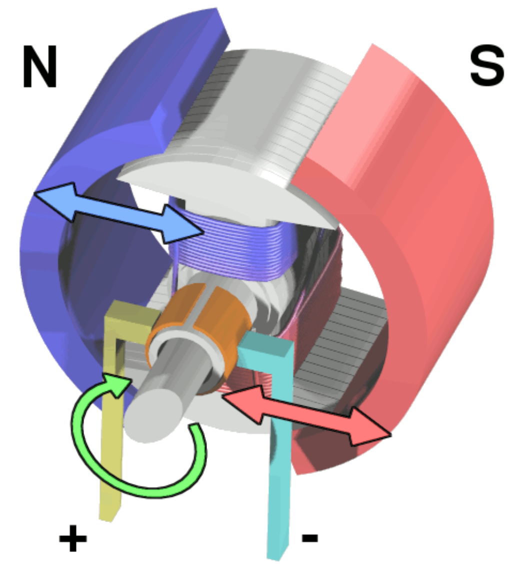
http://micro.magnet.fsu.edu/electromag/java/generator/ac.html



DC Generator





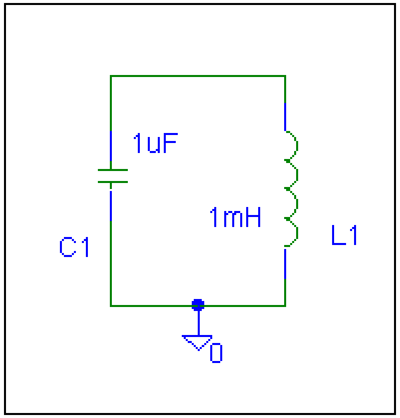


[http://micro.magnet.fsu.e
generator/dc.html](http://micro.magnet.fsu.e)

What Happens if we Combine L & C

- We can trade energy back and forth between the two components, so a periodic oscillation will occur
- If we start with all of the energy in the capacitor and then connect it to an inductor, it will go back and forth forever, unless there is some resistance in the circuit.
- All inductors have some resistance, so the oscillation will die down after some time.

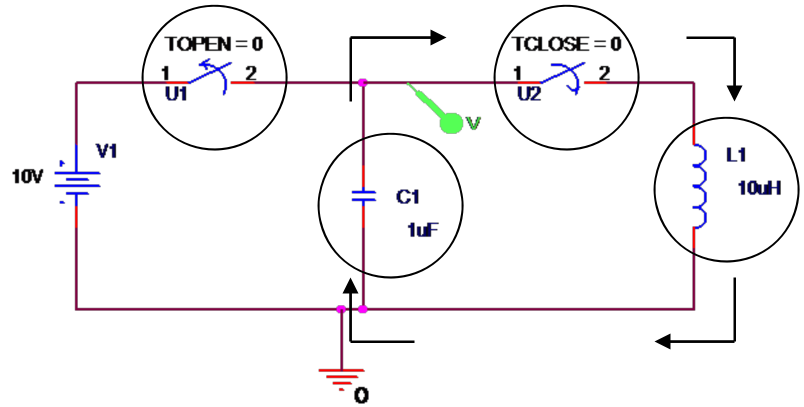
Tank Circuit: A Classical Method Used to Produce an Oscillating Signal



- A Tank Circuit is a combination of a capacitor and an inductor
- Each are energy storage devices

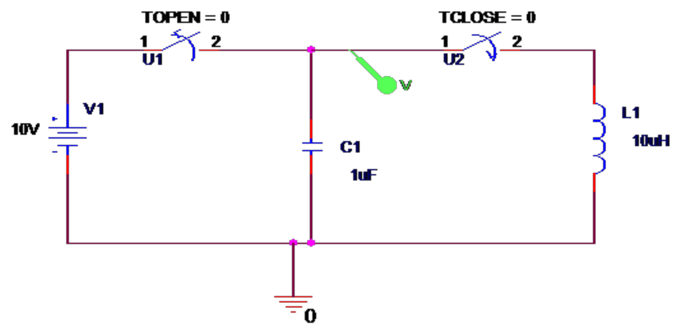
$$W_M = W_L = \frac{1}{2} L I^2 \qquad W_E = W_C = \frac{1}{2} C V^2$$

How Does a Tank Circuit Work?



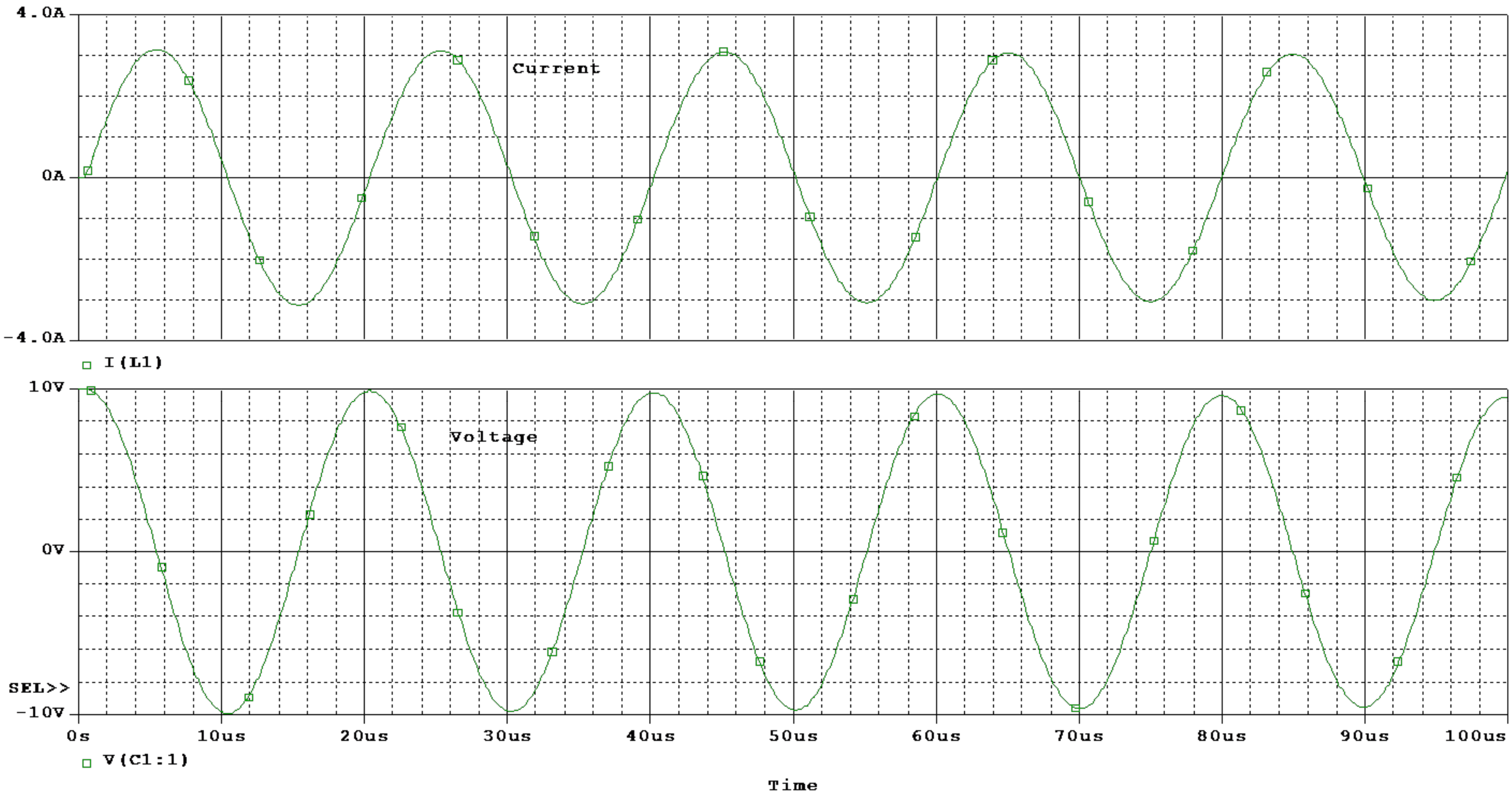
- Charge capacitor to 10V. At this point, all of the energy is in the capacitor.
- Disconnect voltage source and connect capacitor to inductor.
- Charge flows as current through inductor until capacitor voltage goes to zero. Current is then maximum through the inductor and all of the energy is in the inductor.

Tank Circuit

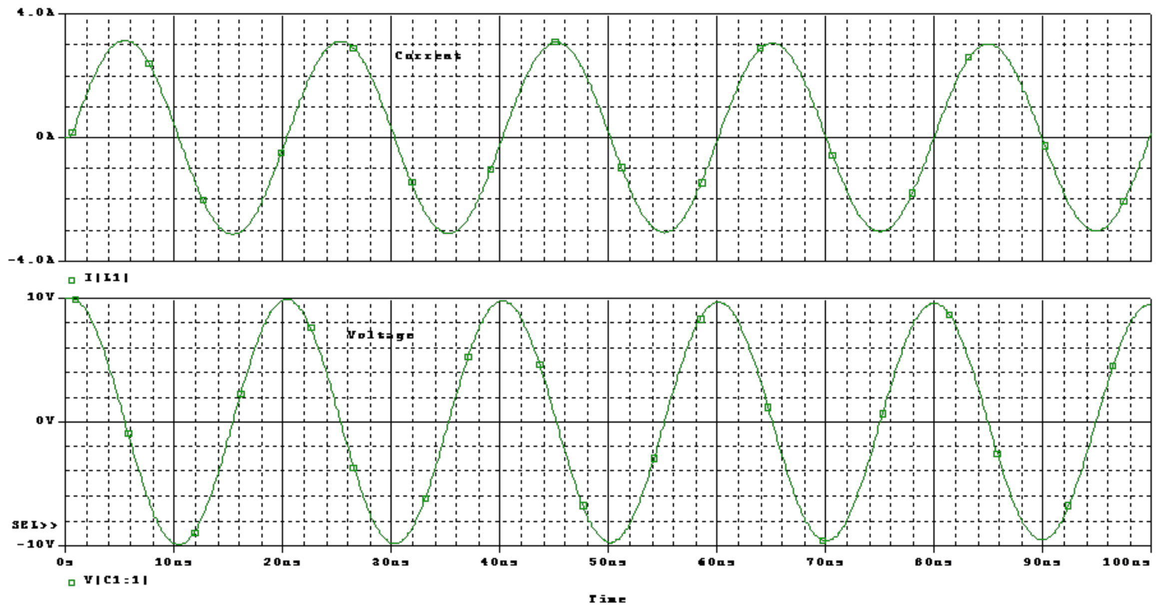


- The current in the inductor then recharges the capacitor until the cycle repeats.
- The energy oscillates between the capacitor and the inductor.
- Both the voltage and the current are sinusoidal.

Tank Voltage & Current



Tank Voltage & Current



- There is a slight decay due to finite wire resistance.
- The frequency is given by $$f = \frac{1}{2\pi\sqrt{LC}}$$

(period shown is about 10ms)